

Evaluate the surface integral

1) $\iint_S x^2 yz \, dS$, S is the part of the plane $z = 1 + 2x + 3y$ that lies above the rectangle $[0, 3] \times [0, 2]$.

2) $\iint_S xy \, dS$, S is the triangular region with vertices $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 2)$.

3) $\iint_S yz \, dS$, S is the part of the plane $x + y + z = 1$ that lies in the first octant.

4) $\iint_S x^2 z^2 dS$, S is the part of the cone $z^2 = x^2 + y^2$ that lies between the planes $z = 1$ and $z = 3$.

5) $\iint_S z dS$, S is the surface $x = y + 2z^2$, $0 \leq y \leq 1$, $0 \leq z \leq 1$.

6) $\iint_S xy dS$, S is the boundary of the region enclosed by the cylinder $x^2 + z^2 = 1$ and the planes $y = 0$ and $x + y = 2$

7) $\iint_S (x^2z + y^2z) dS$, S is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$

8) $\iint_S \sqrt{1+x^2+y^2} dS$, S is the helicoid with vector equation $\vec{r}(u, v) = u \cos v \mathbf{i} + u \sin v \mathbf{j} + v \mathbf{k}$, $0 \leq u \leq 1$, $0 \leq v \leq \frac{\pi}{2}$.

Evaluate the surface integral $\iint_S \vec{F} \cdot d\vec{S}$ for the given vector field \vec{F} and the oriented surface S . In other words, find the flux of \vec{F} across S . For closed surfaces, use the positive orientation.

9) $\vec{F}(x, y, z) = xy \mathbf{i} + 4x^2 \mathbf{j} + yz \mathbf{k}$, S is the surface $z = xe^y$, $0 \leq x \leq 1$, $0 \leq y \leq 1$, with upward orientation.

10) $\vec{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z^4\mathbf{k}$, S is the part of the cone $z = \sqrt{x^2 + y^2}$ beneath the plane $z = 1$ with downward orientation.

11) $\vec{F}(x, y, z) = y\mathbf{j} - z\mathbf{k}$, S consists of the paraboloid $y = x^2 + z^2$, $0 \leq y \leq 1$, and the disk $x^2 + z^2 \leq 1$, $y = 1$.

12) $\vec{F}(x, y, z) = x\mathbf{i} + 2y\mathbf{j} + 3z\mathbf{k}$, S is the cube with vertices $(\pm 1, \pm 1, \pm 1)$.

13) The temperature at the point (x, y, z) in a substance with conductivity $K = 6.5$ is $u(x, y, z) = 2y^2 + 2z^2$. Find the rate of heat flow inward across the cylindrical surface $y^2 + z^2 = 6$, $0 \leq x \leq 4$.